No. 998

INERTIA OF DYNAMIC PRESSURE ARRAYS

By Hans Weidemann

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL MEMORANDUM NO. 998

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There is a certain time lag in dynamic pressure changes before they are visible to the pilot on the indicator dial. This lag depends upon the size of the employed tubing and the characteristics of the indicating instrument. A new mathematical term "pneumatic time constant," which depends only on the dimensions of the tubing and of the indicator, enables the comparison of different dynamic pressure arrays. The pneumatic time constant is an indication of the inertia of a dynamic pressure array.

SUMMARY

From earlier measurements and the mathematical examples, it can be gathered that the inertia of dynamic pressure arrays can be computed with sufficient accuracy and the proper size of tubing established, provided that certain requirements are made on the inertia.

A formula is also given by which the inertia can be computed. Similar in structure to formula $t' = R* \, C*$, it reads as follows:

$$t' = const \; \frac{l}{r^4} \; V \; \frac{\eta}{\gamma} \; \frac{1}{T}$$

$$= 0.087 \; \frac{l}{r^4} \; V \; \frac{\eta}{\gamma} \; \frac{1}{T} \quad (s) \; (reference \; 1)$$

The inertia which indicates the time interval during which a pressure fluctuation has abated to the eth part of its initial amplitude makes it possible to compare different dynamic pressure arrays without regard to eventual diversity of tubing length, diameter, or volume of air-speed indicator.

1. INTRODUCTION

The basic data of the present report are the measurements made by the author in 1936 at the Institute of Technology at Braunschweig. Earlier measurements are checked by calculation and extended.

The indication of the speed of aircraft relative to the air is afforded from measurement of the dynamic pressure $q = \frac{1}{2} \rho v^2$. A nozzle or pitot connected by tubing with the air-speed indicator in the form of an aneroid capsule serves as pickup.

The speed of the air relative to the ground is disregarded since it plays no part in the following arguments and investigations.

If, for any reason, such as the result of a gust, a speed change Δv occurs at the test point, the dynamic pressure Δq changes also. The pressure change is continued through the tubing, a positive pressure change $+\Delta q$ causing an inflow, and a negative pressure change $-\Delta q$ an outflow of a certain volume of air.

It amounts to $\pm \Delta V = \pm \dfrac{\Delta q\ V}{p_0}$, if V is the volume of the total dynamic pressure array and p_0 is the static pressure of air. This volume of air requires a certain time to flow into or out of the dynamic pressure array. Even if the aneroid capsule with its indicator system follows the pressure change practically without inertia (it is assumed to be so small as to be practically negligible), a certain time still elapses before the pointer registers the new dynamic pressure $(q + \Delta q)$. This inertia is defined and computed.

2. DEFINITION OF INERTIA

a) <u>Pneumatic and electric analogy</u>.- The usual dynamic pressure array consists of pickup, tubing, and indicator dial (fig. 1). To make the concept of inertia comprehensible, the dynamic pressure array is replaced by a similar electric system; pitot and connecting tubing correspond electrically to an ohmic resistance R, the aneroid capsule to a condenser C, the dynamic pressure q to voltage u.

When a charged condenser (fig. 2) of capacitance C discharges across a resistance R, the decrement of the condenser voltage u follows according to an exponential function. The same holds true for the pressure in the aneroid capsule by equalization through the connecting tubing. In other words, it involves an equalizing process for which a pneumatic time constant can be given, which is an indication for the rate of decrement of charge or discharge of a condenser. The pneumatic time constant is, in its structure, exactly like the electric time constant and indicates the time rate in which the test quantity has decreased to its eth part, or the time rate after which it has built up to $1/e$ of its final value. This time constant is therefore an indication of the inertia of the compensating process and hence is defined as "the inertia." Viewed electrically, the formula for the inertia reads:

$$t' = R\, C \quad (s)$$

With R* as the flow resistance of the tubing, and C* as the capacitance of the aneroid capsule, "the inertia" of the dynamic pressure system is

$$t' = R*\, C* \quad (s) \tag{1}$$

b) <u>The calculation of R* is based upon the following argument</u>.- Applying a voltage difference Δu to an ohmic resistance R, a current i of magnitude $i = \Delta u/R$ passes through it according to the ohmic law. On applying a pressure difference Δp to a tubing, its flow volume per second can be computed from the Hagen-Poiseuille law. It is

$$Q = \frac{\pi\, r^4}{8\, \eta\, l}\, \Delta p$$

This formula, wherein volume Q (per second) corresponds to current i, and the pressure difference Δp to the voltage difference Δu, is built up exactly like the formula $i = u/R$; hence, $\pi r^4/8\eta l$ defines a reciprocal resistance $1/R$. The flow resistance in the tubing, accordingly, is:

$$R* = \frac{8\eta l}{\pi r^4} \left[\frac{kg\ s}{m^5} \right] \tag{2}$$

where l is the length, r the radius of tubing, and η the viscosity of flow medium.

c) The capacity C* is defined as the capacity of any body of volume V by a pressure change dp; it follows from the general gas equation at

$$C* = - \frac{dV}{dp}$$

and affords, after differentiation:

$$- \frac{dV}{dp} = \frac{V_0^\kappa \, p_0}{V^{\kappa-1} \, \kappa \, p^2}$$

Assuming that p differs little from p_0, which is permissible at pressures around 100 mm Hg, we can put $V_0 = V$. Then the capacity is

$$C* = - \frac{dV}{dp} = \frac{V_0}{\kappa \, p_0} \left[\frac{m^5}{kg} \right] \qquad (3)$$

When computing the capacity of the aneroid capsule, it should be borne in mind that the capsule deflects outward and thus is able to take up a greater volume of air than it otherwise could do. Hence the volume of inflow ΔV due to the pressure change Δp is augmented by a further volume $\Delta V'$ due to the flexibility of the aneroid (fig. 3). Now the substitute volume V* which the aneroid box should have if the total volume of inflow were $(\Delta V + \Delta V')$ without its walls being flexible, can be computed.

It amounts to

$$V* = \left(\frac{\Delta V + \Delta V'}{\Delta p} \right) p_0$$

Thus the flexibility of the aneroid wall causes an increase in capacity C*.

$$C* = - \frac{dV*}{dp} = \frac{V*}{\kappa \, p_0}$$

This increase is especially pronounced on the annular balance where the flexible wall forms the sealing liquid.

d) A third quantity, which electrically corresponds with the inductance L* and appears chiefly by rapidly and

periodically changing pressures, is the mass inertia of the flowing medium in the tubing. It is proportional to its density and amounts to:

$$L^* = \frac{\rho \, l}{F} = \frac{\rho \, l}{r^2 \pi} \left[\frac{kg \, s^2}{m^5} \right]^{1)}$$

(4)

It is usually so small as to be negligible. The respective time constants L^*/R^* and $R^* C^* = t'$ follow logically in seconds.

3. CRITICAL ANALYSIS

In the foregoing arguments certain simplifying assumptions were unavoidable. Thus, it was assumed that the tubing has a resistance but no capacity. This is permissible for narrow tubing whose capacity is unusally low while the resistance is high; on wide tubing the conditions are reversed. Most dynamic pressure indicators are provided with a damping capillary, so that its resistance must also be taken into account.

The complete substitute system is achieved by connecting the tube resistance with that of the damping capillary in series; and that of the capacity of the aneroid capsule and the tubing capacity in parallel (fig. 4).

Then

$$R^* = R_l + R_d \quad \text{and} \quad C^* = C_l + C_d$$

Hence

$$t' = R^* C^* = (R_l + R_d)(C_l + C_d)$$

Accordingly, three cases are possible:

1. Narrow pipe:

$$R_l > R_d \quad \text{and} \quad C_l < C_d$$

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[1] For the definition of L^* and C^*, see reference 1. The definition of R^* is taken from an unpublished report by W. Kerris, Braunschweig. (See also reference 2.)

that is, the effective capacity is essentially that of the aneroid box, while the resistance of the tubing exceeds that of the damping capillaries. Then

$$t' = R_l C_d$$

2. Medium tubing:

$$R_l = R_d \quad \text{and} \quad C_l = C_d$$

that is, capacity and resistance are approximately of the same order of magnitude. Then

$$t' = R^* C^* = 4 R_d C_d = 4 R_l C_l$$

3. Wide tubing:

$$R_l < R_d \quad \text{and} \quad C_l > C_d$$

that is, the capacity of the tubing exceeds that of the aneroid capsule, while the resistance of the tubing can be disregarded with respect to that of the damping capillaries. Then

$$t' = C_l R_d$$

a) Optimum Tubing

The question of optimum tubing with a minimum inertia for a given aneroid capacity and damping resistance can be settled by differentiation of the equation

$$t' = (R_l + R_d) (C_l + C_d)$$

Differentiation of t' with respect to r and posting $r^2 = x$, affords a cubic equation

$$x^3 - \frac{8 \eta l}{\pi R_d} x - \frac{16 \eta \kappa p_o C_d}{\pi^2 R_d} = 0 \quad \text{or} \quad x^3 - 3px - 2q = 0$$

whereby

$$p = \frac{8 \eta l}{3 \pi R_d} \quad \text{and} \quad q = \frac{8 \eta \kappa p_o C_d}{\pi^2 R_d}$$

which contains one real and two conjugate complex solutions; from the real solution follows a tube radius for which the time constant is a minimum.

To illustrate: For

$$R_d = 62 \times 10^6 \frac{kg \cdot s}{m^5} \quad \text{and} \quad C_d = 44.5 \times 10^{-10} \frac{m^5}{kg}$$

we obtain an optimum radius of $r = 1.22 \times 10^{-3}$ meters, with a time constant of 0.406 second. For a tube radius twice as great, $r = 2.44 \times 10^{-3}$, the time constant is 0.61 second; for half the radius, $r = 0.61 \times 10^{-3}$ meters, it was 0.945 second.

The best tube radius therefore is:

$$r_{opt} = \sqrt{x} = \sqrt{2 \sqrt{p} \cosh \frac{\varphi}{3}} \quad \text{whereby} \quad \varphi = \frac{q}{p\sqrt{p}}$$

To illustrate the effect of inertia on a process, let us assume that the process taking place at the test point follows the course of a rectangular function. According to figure 5, the amplitude of the process registers correctly on the indicator only when it continues for at least five time constants ("inertias"). After about five time constants, the amplitude has approached the final value to within about 0.67 percent, that is, has practically reached it. If the duration of the process was less than five time constants, the amplitude is correspondingly false. It falls so much shorter of the maximum value as the particular process is shorter.

The relation that can be derived from figure 5 for the amplitude by known time constant t' and given duration of the process t is:

$$A_t = A_{max} \left(1 - \frac{1}{e^n}\right)$$

if $e = 2.71828$ and t is measured in n multiples of t. Consequently,

$$A_{max} = \frac{e^n}{e^n - 1} A_t$$

b) Wave Equation

For exact procedure, the capacity C must be assumed
as evenly distributed over the whole tubing, and likewise
the resistance R; the inductance L can usually be as-
sumed negligibly small; the derivation G itself — elec-
trically the reciprocal of an insulation resistance, pneu-
matically the porosity of the tubing - is equated to zero.
Then if the tubing alone is considered, the wave equation
reads:

$$\frac{\partial^2 p}{\partial x^2} = L_1\ C_1\ \frac{\partial^2 p}{\partial t^2} + R_1\ C_1\ \frac{\partial p}{\partial t}$$

(L_1, R_1, and C_1 refer to length unit.) Figure 6 illus-
trates the electric substitute scheme at the tubing. Such

a tubing has the wave resistance $Z = \sqrt{L_1/C_1}$, and the
pressure waves advance over it at the rate of $v = 1/\sqrt{L_1 C_1}$.
The speeds of advance are all at 338 meters per second, as
shown in table I, that is, velocity of sound. The resist-
ance of the tubing results in exponential damping of the
pressure amplitude and amounts to

$$p = p_0\ e^{-\frac{R_1 x}{2\ Z}}$$

The damping of the pressure amplitude in percent
amounts to 72 percent for the narrowest tubing
($r = 0.75 \times 10^{-3}$ m), and only 3 percent for the widest
($r = 5 \times 10^{-3}$ m), on the assumption that only one single
pressure wave is involved. Then one may no longer speak
of a pneumatic time constant because it is dependent upon
the locality, hence the speed of propagation must be ac-
counted for along with the exponential damping of the
pressure amplitude.

But for practical purposes, the discussed method is
sufficient if it involves, for instance, the dimensions
of length and diameter of the tubing by given air-speed-
meter dimensions, so as to maintain the inertia within
certain defined limits.

TABLE I

		$r=0.75 \times 10^{-3}$	$r=3 \times 10^{-3}$	$r=5 \times 10^{-3}$
R_1	$\dfrac{kg\ s}{m^6}$	14.7×10^6	0.575×10^6	0.00745×10^6
L_1	$\dfrac{kg\ s^2}{m^6}$	0.0695×10^6	0.00435×10^6	0.00156×10^6
C_1	$\dfrac{m^4}{kg}$	1.26×10^{-10}	20.2×10^{-10}	56.2×10^{-10}
R_l	$\dfrac{kg\ s}{m^5}$	59×10^6	0.23×10^6	0.0298×10^6
L_l	$\dfrac{kg\ s^2}{m^5}$	0.278×10^6	0.0174×10^6	0.00624×10^6
C_l	$\dfrac{m^5}{kg}$	5.05×10^{-10}	80.8×10^{-10}	224.8×10^{-10}
Z	$\dfrac{kg\ s}{m^5}$	23.4×10^6	14.7×10^6	0.527×10^6
v	m/s	338	338	338
t_1	s	0.0118	0.0118	0.0118
$\dfrac{R_l}{2Z}$	1	1.26	0.0782	0.0283
$\dfrac{p_0 e^{-\frac{R_l}{2Z}}}{p_0}$	$\%$	72	8	3

$$\kappa = 1.4 \quad p_0 = 10^4\ kg/m^2 \quad \eta = 1.83 \times 10^{-6}\ \frac{kg\ s}{m^2}$$

$$\rho = 0.123\ \frac{kg\ s^2}{m^4} \quad l = 4\ m$$

$$C^*_{airspeed\ meter} = -\frac{dV^*}{dp} = \frac{V^*}{\kappa p_0}$$

$$\Delta V = \frac{\Delta p}{p_0} V = \frac{200 \times 12.31 \times 10^{-6}}{10^4} = 0.246 \times 10^{-6}\ m^3$$

$$\Delta V^*_{measured} = 1 \times 10^{-6}\ m^3 \quad V = 12.3 \cdot 10^{-6}\ m^3$$

$$\Delta V + \Delta V' = (1 + 0.246) \times 10^{-6} = 1.246 \times 10^{6} \quad m^{3}$$

$$V^{*} = \frac{p_{o}}{\Delta p} (\Delta V + \Delta V') = \frac{1.246 \times 10^{-6} \times 10^{4}}{200} = 62.3 \times 10^{-6} \quad m^{3}$$

$$C^{*} = \frac{V^{*}}{\kappa p_{o}} = \frac{62.3 \times 10^{-6}}{1.4} 10^{-4} = 44.5 \times 10^{-10} m^{5}/kg$$

4. CALCULATION AND MEASUREMENT OF THE INERTIA

a) Testing Equipment (fig. 7)

A 12-liter equalizing tank is put under low pressure by means of an ejector water pump and read on a water manometer. The opening of a cock fitted with pawl and a heavy spring under initial tension is accompanied by a low-pressure wave through the tubing and deflects the pointer of a commercial Askania air-speed meter fastened at the end of the tubing. The passage of the pointer through certain marks is observed by telescope and the time clocked by stop watch.

b) Measurement

The measurement was based on the following reasoning: An airplane flies with a certain dynamic pressure and is struck by a sudden gust. This causes a sudden rise in dynamic pressure. Then the pilot is very much interested in knowing the time lapse before this pressure change registers on the air-speed meter. The low pressures produced ranged from 10 to 80 millimeters of mercury, corresponding to an air-speed meter reading of from 100 to 250 kilometers per hour; then the time interval after which the pointer deflection had dropped to the eth part of its initial reading was recorded.

For other reasons, to be explained later, only the measurements at 10 mm Hg $\approx$ 100 km/h were taken into account and the time interval recorded - after which the air-speed-meter deflection had dropped to 63 kilometers per hour.

Since the inertias were so small that clocking by stop watch seemed useless, artificial enlargement was necessary. This was achieved with the capillaries attached to the

front end of the tubing. The total resistance of the tubing then followed from the series arrangement of the resistance of the capillary and that of the tube, and the total capacity from a parallel arrangement of the capacity of the tube and that of the air-speed meter.

The resistances of the damping capillaries, numbered for reasons of simplicity, are appended in table II.

TABLE II

Capillary no.	r (m)	R_{cap} (kg s/m^5)
VI	0.30×10^{-3}	115×10^6
V	0.345×10^{-3}	65.6×10^6
IV	0.35×10^{-3}	62.0×10^6
III	0.355×10^{-3}	58.7×10^6
II	0.45×10^{-3}	22.7×10^6

Length of each capillary: 0.2 m

As stated in the foregoing, the calculation of the capacity had been based upon a pressure difference of 100 millimeters W.S. $= \frac{1}{100}$ atmosphere. At higher pressures, the capacity is no longer a constant but a function of the pressure; likewise, the inductance L^* is no longer constant because of the dependence of the density ρ on the pressure, and the resistance R^* itself ceases to follow Poiseuille's law, since the flow is no longer laminar. For this reason the measurements at pressures from 20 to 80 millimeters of mercury were for the present not evaluable — thus leaving only the test data at 10 millimeters of mercury amenable to a mathematical check.

c) Results

Table III contains the mathematical and experimental data of the inertia for different tube diameters and different damping capillaries.

TABLE III

Pressure　p = 10 mm Hg,　air-speed-meter reading: 100/63
km/h,　inertia　t'(s)

Capillary no.	r=5×10⁻³ m		r=3×10⁻³ m		r=0.75×10⁻³ m	
	Experimental	Theoretical	Experimental	Theoretical	Experimental	Theoretical
VI	2.85　2.85	2.98	1.55	1.45	0.9	0.775
V	1.95　2.0	1.96	1.2	.825	.85	.555
IV	1.8　1.8	1.86	1.05	.78	.8	.54
III	1.45　1.5	1.75	1.0	.74	.75	.525
II	1.25　1.2	.61	.95	.288	.75	.405

The agreement between the computed and the erstwhile
recorded values is good.

d)　Ju 52

For further example, take the inertia of the dynamic
pressure array of a　Ju 52 airplane.　For a tube 10 meters
long, with a　6×10⁻³ millimeter diameter, the resistance
of the line　R^*　amounts to　0.573×10⁶ kg s/m⁵;　the capacity　C^*,　to　202×10⁻¹⁰ m⁵/kg.

The resistance of the tube relative to that of the
capillaries is disregarded, but the capacity exceeds that
of the air-speed meter; both being parallel,　C^*_{tot}　amounts
to　246.5×10⁻¹⁰ m⁵/kg.

TABLE IV

Capillary no.	R_{cap} (kg s/m⁵)	t' theoretical (s)	t' experimental (s)
II	23.27 × 10⁶	0.585	1.0
III	59.35 × 10⁶	1.46	1.3
IV	62.57 × 10⁶	1.54	1.4
V	66.17 × 10⁶	1.63	1.5
VI	115.57 × 10⁶	2.85	2.35

Here also satisfactory agreement is obtained. On the earlier measurement the volume of the tube was enlarged to 284×10^{-6} m^3 by insertion of a suitable glass vessel.

e) Elbows or Bends

The earlier study included straight tubing as well as the effect of bends. But none of the tests made in this direction disclosed effects of curvature in the tubing on the inertia of the record with the means available.

f) Pitot Tubes

A complete dynamic pressure array includes the pickup, that is, the pitot tube itself. Hence the study included tubing with a pitot tube at the end. Prandtl's needle pitot tube manifested an increased inertia of several tenths of seconds; all others failed to disclose any increase. Thus, it follows that no great weight attaches to the resistance of the pitot tube and consequently it does not affect the inertia. The studies on pitot tubes disclosed that no resistance in the sense of a tubing should be coordinated to them since sharp bends, contractions, etc., produce additional pressure gradients within the pitot tubes, which is equivalent to an increase in resistance. As in the electric measurement of the resistance through current and voltage measurements, the pressure drop and the flow volume of air were recorded by different pitot tubes and then the same measurement repeated on tubing of the same length and diameter as those of a pitot tube. The ratio of both resistances was defined as the quality factor k of a pitot. This factor indicates how many times greater the resistance of an extended length of tubing must be than that of a pitot in order to produce by the same flow volume per second the same pressure drop as the latter. At Reynolds numbers of from 600 to 16,000 the quality factor k, ranging between 1.3 and 1.6 for most pitots - was found to be constant.

REFERENCES

1. Trendelenburg, F.: Einführung in die Akustik.
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2. Oppelt, W., and Wenke, F.: Theoretische Betrachtung
      des Luftdruckvariometers und Richtlinien für Ent-
      wurf und Prüfung. Luftfahrtforschung, Bd. 14,
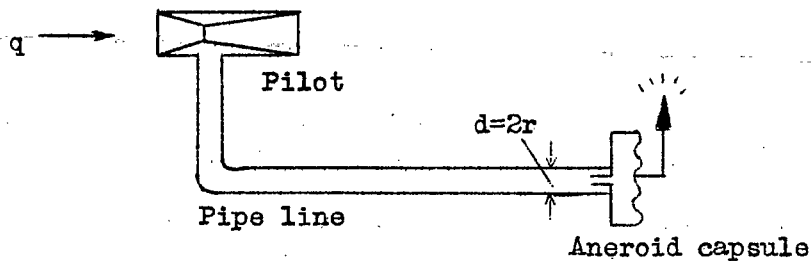   Lfg. 10, Oct. 12, 1937.

$q \longrightarrow$

Pilot

d=2r

Pipe line

Aneroid capsule

Figure 1.-   Diagrammatic sketch of a
dynamic pressure array.

R

u

C
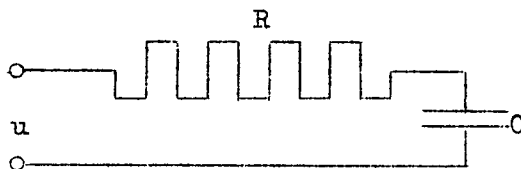
Figure 2.-   Charge of a condenser over a resis-
tance (simplified electric substitute
diagram for a dynamic pressure array).

$\Delta V'$

ΔV
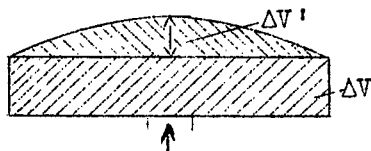
Figure 3.-   Diagrammatic presentation of the
additional inflow volume V' result-
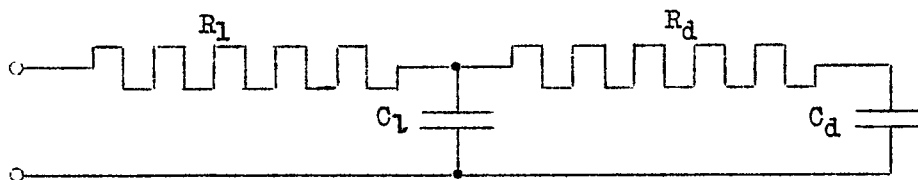ing from the flexibility of the aneroid
wall.

R_1

R_d

C_1

C_d

Figure 4.-   Electrical substitute diagram of a
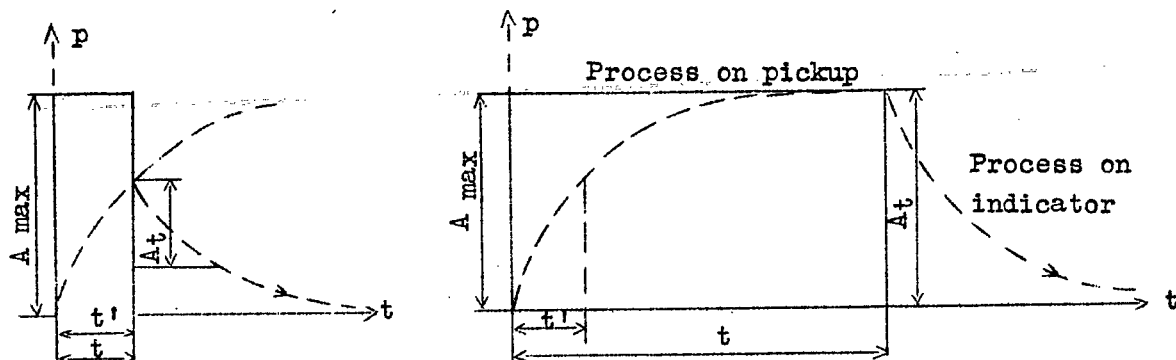dynamic pressure array.

Figure 5.-   Curve of two dissimilar rectangular dynamic pressure
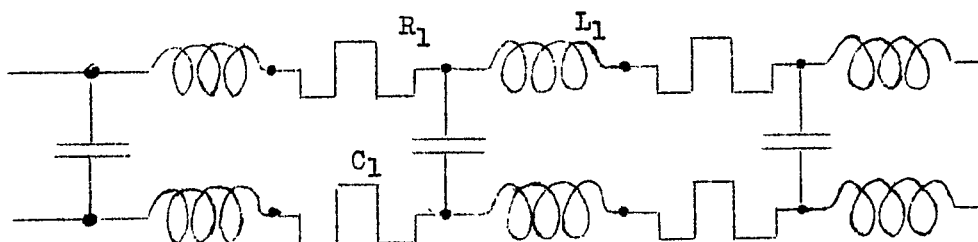   arrays on the pickup and on the indicator dial.

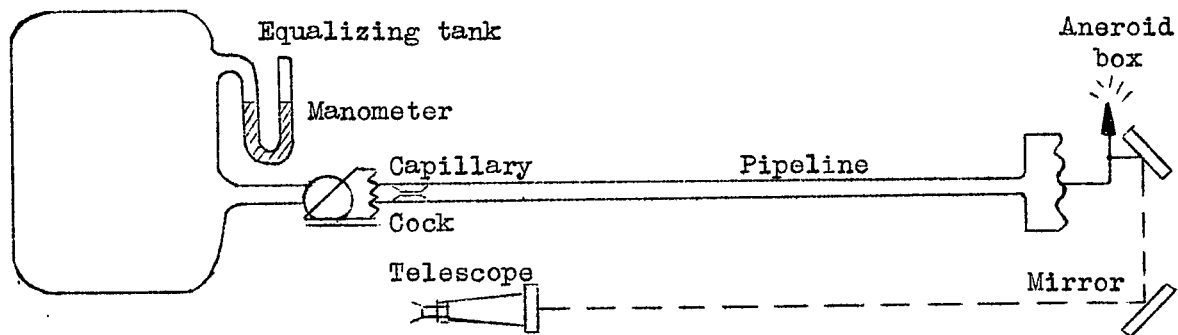Figure 6.-   Electric substitute diagram of a tubing.

Figure 7.-   Diagrammatic sketch of test equipment.

N O T I C E

The Committee's Technical Memorandum No. 998, entitled "Inertia of Dynamic Pressure Arrays," by Hans Weidemann has been carefully read by Professor K. J. DeJuhasz, of the Pennsylvania State College, and he has pointed out some errors in the original text. Several other typographical errors have also been discovered in this Technical Memorandum. Some of these errors were in the original and others were in the translation.

An errata sheet has been prepared indicating all the errors found, and a copy of the errata is attached hereto for insertion in your copy of the Technical Memorandum.

Page 4, line 6, the equation should read:

$$- \frac{dV}{dp} = \frac{V_0^{\kappa} \, p_0}{V^{\kappa - 1} \, \kappa \, p^2}$$

Page 6, fourth line from bottom, equation should read:

$$r^2 = x$$

Page 7, last two equations should read:

$$A_t = A_{max} \left(1 - \frac{1}{e^n} \right)$$

$$A_{max} = \frac{e^n}{e^n - 1} A_t$$

Page 8, second centered equation should read:

$$p = p_0 e^{-\frac{R_1 x}{2Z}}$$

Page 9, table I, fourth column, top row should read:

$$0.0575 \times 10^6$$

fifth column, sixth row should read:

$$224.8 \times 10^{-10}$$

first column, last row should read:

$$\frac{p_0 e^{-\frac{R_l}{2Z}}}{p_0}$$

E R R A T A (Cont.)

Page 9, in second line below table:

"1 = 4m"   should read   "l = 4m"

Page 9, in last line:

"$V'_{measured}$"   should read   "$\Delta V^*_{measured}$"

Page 10, in line 2:

"$\dfrac{p_o}{p}$"   should read   "$\dfrac{p_o}{\Delta p}$"

Page 11, table II, column 2, line 1, should read:

0.30 × 10^{-3}